

**ICS test part 1, 17-12-2013**

No open book.

Language Dutch or English.

Duration 60 minutes.

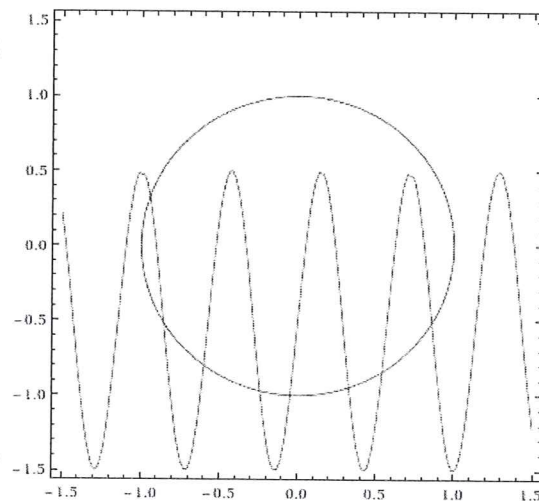
1: A system of equations  $Ax=b$  is solved by LU decomposition and by forward substitution and backward substitution, where  $A$  is an  $N \times N$  matrix.

1A: What is the time complexity of decomposing  $A$  in its  $L$  and  $U$  triangular matrices. (2 pts.)

1B: Explain the process of backward substitution. (4 pts.)

1C: Of a matrix  $A$  the LU decomposition is given. Using this decomposition, how should the inverse of  $A$  be computed? What is the time complexity? (4 pts.)

2A: A disk of radius 1.0 is defined by the expression  $x^2 + y^2 - 1.0 = 0$ . Also a sine curve  $y = \sin(11x) - 0.5$  is given. We want to approximate numerically the area defined by the intersection of the circle and the part below the sine curve by using random numbers, see figure. Write an algorithm (in pseudo code) for this purpose. Assume that you have a function  $random(f1, f2)$  which returns a floating point random number between the floating point boundaries  $f1$  and  $f2$ . (The pseudo code algorithm will be 10 lines of code or less.) (8 pts.)



2B: How can you give an estimate of the precision of the calculated area. (2 pts)

3A: An M.D. simulation of a system of  $N$  particles is performed. What is the time complexity of the most simple implementation of this simulation? Explain. (3 pts.)

3B: Give two techniques to speed up an M.D. simulation. (3 pts.)

3C: In the M.D. context, explain the notion of periodic boundary conditions. (4 pts.)

## Test ICS 2013-2014, Part 2

No open book.

Duration 30 min

### 4. Ordinary Differential Equations

- A. Given a time-independent ordinary differential equation

$$\frac{d\vec{x}}{dt} = f(\vec{x}) \quad (1)$$

give the condition for equilibrium, and given an equilibrium point  $\vec{x}_0$  describe how to determine the stability of this equilibrium (5pts)

- B. What is the simplest numerical method to solve an ordinary differential equation (assume it is an initial value problem). What are the drawbacks of this simple method. (5pts)

### 5. Partial Differential Equations

- A. Give one example of a partial differential equation and describe how the concept differs from an ODE. (5pt)
- B. Given a problem in which we expect both very smooth areas and areas with sudden jumps in the solution to a PDE, which would be more suitable: Finite Element Method or Finite Difference Method? Motivate your answer. (5pt)

### 6. Game Theory

- A. What is a pay-off matrix, and what does it describe? (3pt)
- B. What is an evolutionary stable strategy? (4pt)
- C. What game theoretical method can be used to model a situation where the two players have very different roles, and therefore very different goals? (3pt)